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Plasma radiation spectra in the presence of static electric and high-frequency radiation fields

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Abstract. Harmonics generation of high-frequency radiation in a plasma embedded in a constant electric field is investigated theoretically. It is shown that the electron directed motion due to the static electric field yields the appearance in the plasma emission spectrum of high-frequency radiation even harmonics. The conditions are established when the even harmonics generation is as effective as that of the odd ones. At variance with the odd harmonics, the even harmonics polarization plane is found to rotate with respect to that of the fundamental field. The basic dependencies concerning the rotation angle and the generation efficiency on the plasma and field parameters are established.

PACS. 52.50.Jm Plasma production and heating by laser beams (laser-foil, laser-cluster, etc.) – 52.38.Dx Laser light absorption in plasmas (collisional, parametric, etc.)

1 Introduction

Radiation spectra are a powerful tool to investigate plasma properties (see, for instance, [1]). If the plasma interacts with a strong high-frequency radiation field the spectral composition of its radiation emission becomes considerably reacher. The reason of such an enrichment has a two-fold nature. First, under the action of strong high-frequency fields in the plasma the development of parametric instabilities is possible [2,3], which contributes to the modification of the radiation spectra. Second, in the radiation spectra may appear harmonics of the high-frequency field itself. In a homogeneous fully ionized plasma one of the basic mechanisms to generate harmonics is given by the electron-ion (e-i) collisions. As shown in paper [4], in the above plasma, e-i collisions in a strong high-frequency field yield generation of odd harmonics only. Absence of even harmonics is consequence of the invariance of the Rutherford e-i scattering cross-section under the symmetry operation of the electron velocity direction inversion, and of the isotropy of the initial Maxwellian electron velocity distribution function (EDF). Generation of only odd harmonics takes place also in the case when the plasma exhibits a bi-Maxwellian EDF [5], which may be formed as a result of the laser radiation inverse bremsstrahlung [6,7]. In this case too, invariance under inversion of the electron velocity direction is preserved. At the same time, experimental conditions may take place when the EDF is not symmetric with respect to the inversion of velocity direction. Very frequent is, for instance, the case when a plasma is embedded in an uniform constant electric field $\overrightarrow{E}_0$. In such a case, in the EDF appears an antisymmetric part, which, in particular, determines the current density. It is just a situation which takes place in direct plasma discharges (see, for instance, the review paper [8]). Besides, if the electric field $\overrightarrow{E}_0$ is switched on in a time smaller than the electron mean free path time, and its strength is smaller than the Dreicer critical field, in the initial stage two time intervals may be singled out. For time smaller than the electron mean free path time a freely accelerated electron motion takes place, which is characterized by a linear in time growth of the current density and by an anisotropic correction to the EDF. For times larger than the electron mean free path time, the freely accelerated electron motion evolves into a quasistationary drift motion, characterized by a velocity which is smaller than the thermal velocity. At these later times, the EDF anisotropic part depends on velocity differently as compared to the first time interval.

In this paper we show that the presence of just an antisymmetric term in the EDF yields the high-frequency field even harmonics generation thanks to e-i collisions. As in different time stages of the discharge the EDF anisotropic part exhibits different dependencies on velocity, the even harmonics generation characteristics in such stages will

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be different. It implies that measuring plasma emission at the high-frequency field even harmonics it is possible to obtain information on the evolution of electron current velocity, which is highly significant to check a series of anomalous properties of current-currying plasmas [8]. Having in mind this possibility, it is worth to investigate the even harmonics properties in plasmas with anisotropic EDF. A first step in this direction is done in the present investigation, where the even harmonics generation is studied in the freely accelerated electron motion regime. Below high-order harmonic generation is investigated in the conditions, when the electron drift velocity in the electric field $\overrightarrow{E}_0$ remains small in comparison to the electron thermal velocity v_T. In such conditions, the influence of the constant field $\overrightarrow{E}_0$ on the odd harmonics generation efficiency is small, and the latter remains essentially the same as for $\overrightarrow{E}_0 = 0$. At the contrary, the even harmonics generation efficiency is proportional to $\overrightarrow{E}_0^2$ and may become of the same order of magnitude as that of odd harmonics. Below, the main attention will be paid to the investigation of even harmonics for which we will establish the generation efficiency as function of the laser and plasma parameters.

We establish below that the largest generation efficiency at frequency $2n\omega$, where $n = 1, 2, ...,$ as well as for $(2n+1)\omega$, is obtained when the ratio of the electron quiver velocity amplitude in the high-frequency field v_E to the thermal velocity v_T is approximately $2\sqrt{n}$. We also show that even harmonics are polarized in a plane rotated from that of the fundamental radiation polarization. The value of the rotation angle depends on the harmonic number, the value of the ratio v_E/v_T and the orientation of the field $\overrightarrow{E}_0$ with respect to the high-frequency field $\overrightarrow{E}$ and its wavevector $\overrightarrow{k}$. It is worthy to note that, in the conditions considered below, for the odd harmonics there is no effect of polarization plane rotation. All odd harmonics are polarized in the same plane as the fundamental high-frequency field.

2 High-frequency currents

Let us consider a fully ionized plasma, interacting with a constant electric field $\overrightarrow{E}_0$ and a high-frequency monocromatic electromagnetic field $\overrightarrow{E}\cos(\omega t - \overrightarrow{k}\cdot\overrightarrow{r})$, $\overrightarrow{k}\cdot\overrightarrow{E} = 0$. We assume that the laser frequency ω is considerably larger than the electron plasma frequency $\omega_L = (4\pi e^2 N/m)^{1/2}$, where e and m are, respectively, the electron charge and mass, and N is the electron density. We confine our investigation to the case when both the electron thermal velocity v_T and the quiver velocity amplitude $v_E = |eE/m\omega|$ are much smaller than the speed of light c. In such conditions the frequency ω is related to the wavevector $\overrightarrow{k}$ by the familiar dispersion relation

$$\omega^2 = \omega_L^2 + k^2c^2. \tag{1}$$

Besides, in a Maxwellian plasma, in considering the electron kinetics in the presence of a high-frequency field, is legitimate to neglect the influence of the variable magnetic field as well as the weak nonuniformity in the electron distribution, caused by the finite value of the vector $\overrightarrow{k}$. Within the above assumptions and simplifications, to determine the EDF f we have the equation

$$\frac{\partial f}{\partial t} + \frac{e}{m}\overrightarrow{E_0}\frac{\partial f}{\partial \overrightarrow{v}} + \frac{e}{m}\overrightarrow{E}\cos(\omega t - \overrightarrow{k}\overrightarrow{r})\frac{\partial f}{\partial \overrightarrow{v}} = St\,(f)\,, \tag{2}$$

where the e-i collision integral has the form

$$St(f) = \frac{1}{2}\nu(v)\frac{\partial}{\partial v_i}(v^2\delta_{ij} - v_iv_j)\frac{\partial f}{\partial v_j}. \tag{3}$$

Here $\nu(v) = 4\pi Ze^4 N\Lambda/m^{-2}v_T^{-3}$ is the e-i collision frequency, Z the ion ionization multiplicity, Λ the Coulomb logarithm.

Being interested in time moments smaller than the inverse of the e-i collision frequency, $t < \nu^{-1}$, $\nu = \nu(v_T)$, the collision integral in equation (2) may be taken into account as a perturbation. Neglecting collisions in equation (2) to the zero approximation we have

$$f_0 = f_m\left[\overrightarrow{v} - \overrightarrow{v}_E\sin(\omega t - \overrightarrow{k}\overrightarrow{r}) - e\overrightarrow{E}_0 t/m\right], \tag{4}$$

where $\overrightarrow{v}_E = e\overrightarrow{E}/m\omega$, $f_m(v) = N(2\pi)^{-3/2}v_T^{-3} \times\exp(-v^2/2v_T^2)$ is a Maxwellian EDF. The solution to equation (2) has been written under the assumption that the constant electric field is switched on at the moment $t = 0$. It corresponds to the physical situation when to the plasma, interacting with the high-frequency laser radiation, a constant electric field is applied for a time much smaller than ν^{-1}. Accordingly the relatively simple solution (4) to the kinetic equation is valid for a time interval small as compared to the inverse of the e-i collision frequency. To the distribution (4) corresponds the electron current density

$$\begin{aligned}\overrightarrow{j}_0 &= e\int d\overrightarrow{v}\,\overrightarrow{v}f_0\\ &= eN\overrightarrow{v}_E\sin(\omega t - \overrightarrow{k}\overrightarrow{r}) + \frac{e^2}{m}\overrightarrow{E}_0 Nt.\end{aligned} \tag{5}$$

According to (5), describing the electrons directed motion in the field $\overrightarrow{E}_0$, the electron current velocity grows proportionally to t in so far as $t < \nu^{-1}$. The addition δf to f_0 due to collisions is described by an equation as (2), where in the l.h.s. enters δf, while in the collision integral enters f_0 instead of f.

Taking into account the identity $\int d\overrightarrow{v}\delta f = 0$, meaning absence of contribution by δf to the electron density, from the kinetic equation for δf we find the time derivative of the correction $\delta\overrightarrow{j} = e\int d\overrightarrow{v}\cdot\overrightarrow{v}\delta f$ to the unperturbed current density $\overrightarrow{j}_0$:

$$\begin{aligned}\frac{\partial}{\partial t}\delta\overrightarrow{j} &= e\int d\overrightarrow{v}\,\overrightarrow{v}St(f_0) = ie\nu v_T^3\int\frac{d\overrightarrow{q}}{(2\pi)^3}\overrightarrow{q}\frac{4\pi}{q^2}\\ &\times\int d\overrightarrow{u}\exp\left[i\overrightarrow{q}\overrightarrow{u} + i\overrightarrow{q}\overrightarrow{v}_E\sin(\omega t - \overrightarrow{k}\overrightarrow{r})\right]\\ &\times f_m\left(\overrightarrow{u} - \frac{e}{m}\overrightarrow{E}_0 t\right).\end{aligned} \tag{6}$$

The relation (6) is all what we need to describe harmonics generation of high-frequency current in a plasma with displaced EDF. In absence of the static electric field $\overrightarrow{E}_0 = 0$, equation (6) describes generation of odd harmonics, with frequencies $(2n+1)\omega$, $n = 1, 2, ...$ [4].

If $\overrightarrow{E}_0 \neq 0$, the same relation (6) describes generation of even harmonics $2n\omega$ as well, where $n = 1, 2, ...$ We show it considering a short time interval, when the electron drift velocity is yet small as compared to the electron thermal velocity $|eE_0 t/m| \ll v_T$, but $\omega t \gg 1$. Under these restrictions, in equation (6) is sufficient to keep only the terms linear in the field $\overrightarrow{E}_0$. Then, by integrating in equation (6) over the velocity $\overrightarrow{u}$, we find

$$\frac{\partial}{\partial t}\delta\overrightarrow{j} = -eN\nu v_T^3 \int \frac{d\overrightarrow{q}}{(2\pi)^3}\frac{4\pi}{q^2}\overrightarrow{q}\exp\left(-\frac{1}{2}q^2 v_T^2\right) \times \left\{ 2\sum_{n=0}^{\infty} J_{2n+1}\left(\overrightarrow{q}\,\overrightarrow{v}_E\right)\sin\left[(2n+1)\left(\omega t - \overrightarrow{k}\cdot\overrightarrow{r}\right)\right] + \frac{e}{m}\left(\overrightarrow{q}\,\overrightarrow{E}_0\right)t\left[J_0\left(\overrightarrow{q}\,\overrightarrow{v}_E\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\overrightarrow{q}\,\overrightarrow{v}_E\right)\cos\left[(2n+1)\left(\omega t - \overrightarrow{k}\cdot\overrightarrow{r}\right)\right]\right]\right\}, \tag{7}$$

where $J_n(x)$ is the nth order Bessel function. From equation (7) for the current odd harmonics $(2n+1)\omega$ we have

$$\frac{\partial}{\partial t}\delta\overrightarrow{j}_{odd} = \sum_{n=0}^{\infty}\frac{\partial}{\partial t}\delta\overrightarrow{j}_{2n+1} \times \sin\left[(2n+1)(\omega t - \overrightarrow{k}\cdot\overrightarrow{r})\right], \tag{8}$$

$$\frac{\partial}{\partial t}\delta\overrightarrow{j}_{2n+1} = -\frac{\omega_L^2}{4\pi}\overrightarrow{E}\sqrt{\frac{2}{\pi}}\frac{\nu}{\omega}\frac{8}{\gamma^3}\int_0^{\gamma/2} dz z^2 \times \exp(-z^2)\left[I_n(z^2) - I_{n+1}(z^2)\right], \tag{9}$$

where $\gamma = v_E/v_T$, I_n the nth order modified Bessel function. According to equation (9) the odd harmonics current is directed along the high-frequency field $\overrightarrow{E}$.

A different situation takes place for the current of even harmonics. As the vector $\overrightarrow{E}_0$ has arbitrary orientation with respect to the vectors $\overrightarrow{k}$ and $\overrightarrow{E}$, in the general case the current density vector $\delta\overrightarrow{j}_{even}$ has components along $\overrightarrow{k}$, $\overrightarrow{E}$ and $\overrightarrow{k}\times\overrightarrow{E}$. For harmonics generation is of interest the vortex part of the current density for which $\text{rot}\,\delta\overrightarrow{j}_{even} \neq 0$ or $\overrightarrow{k}\times\delta\overrightarrow{j}_{even} \neq 0$. Using these considerations, for the current density vortex part at frequencies $2n\omega$, $n = 1, 2, ...$, from equation (7) we find

$$\frac{\partial}{\partial t}\delta\overrightarrow{j}_{even} = \sum_{n=1}^{\infty}\frac{\partial}{\partial t}\delta\overrightarrow{j}_{2n} \times \cos\left[(2n+1)(\omega t - \overrightarrow{k}\cdot\overrightarrow{r})\right], \tag{10}$$

$$\frac{\partial}{\partial t}\delta\overrightarrow{j}_{2n} = -\frac{\omega_L^2}{4\pi}\sqrt{\frac{2}{\pi}}\nu t\int_0^1 dx\left\{2x^2\overrightarrow{\varepsilon}(\overrightarrow{\varepsilon}\,\overrightarrow{E}_0) + (1-x^2)\overrightarrow{\kappa}(\overrightarrow{\kappa}\,\overrightarrow{E}_0)\right\}F\left[n, \frac{1}{4}x^2\gamma^2\right], \tag{11}$$

where $\overrightarrow{\varepsilon} = \overrightarrow{E}/E$, $\overrightarrow{\kappa} = \overrightarrow{k}\times\overrightarrow{E}/kE$ and the function $F(n, y)$ has the form

$$F(n, y) = \left[(2n+1-2y)I_n(y) + 2yI_{n+1}(y)\right]\exp(-y). \tag{12}$$

Besides the high-frequency currents equations (8–12), formula (7) contains a term proportional to $J_0(\overrightarrow{q}\cdot\overrightarrow{v}_E)$, which for $\omega t \gg 1$ describes a contribution to the current slowly varying in time. This term is out of interest for the harmonic generation theory, and its discussion in what follows will be omitted.

3 Harmonics field in the plasma

To determine the strength of the field generated in the plasma at the frequency $(2n+1)\omega$, let us use the wave equation with a current source, described by one of the terms of equation (9), which has the form

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial\overrightarrow{r}^2} + \omega_L^2\right)\overrightarrow{E}_{2n+1}(\overrightarrow{r}, t) = -4\pi\frac{\partial}{\partial t}\delta\overrightarrow{j}_{2n+1}\sin\left[(2n+1)(\omega t - \overrightarrow{k}\cdot\overrightarrow{r})\right]. \tag{13}$$

This linear equation has the following forced solution

$$\overrightarrow{E}_{2n+1}(\overrightarrow{r}, t) = -\overrightarrow{E}_{2n+1}\sin\left[(2n+1)(\omega t - \overrightarrow{k}\cdot\overrightarrow{r})\right]. \tag{14}$$

Harmonics generation efficiency η_{2n+1} is defined as the ratio of the energy flux density at the frequency $(2n+1)\omega$, $I_{2n+1} = cE_{2n+1}^2/8\pi$, to that at the fundamental frequency $I = cE^2/8\pi$. Taking into account the dispersion relation of the fundamental wave (1), the relations (9), (13) and (14), for the $(2n+1)\omega$ harmonics generation efficiency we find

$$\eta_{2n+1} = \left(\frac{\nu}{\omega}\right)^2 H(2n+1, \gamma), \tag{15}$$

$$H(2n+1, \gamma) = \frac{8}{\pi}\gamma^{-6}\left\{\frac{1}{n(n+1)}\int_0^{\gamma/2} dz z^2 \times \exp(-z^2)\left[I_n(z^2) - I_{n+1}(z^2)\right]\right\}^2. \tag{16}$$

For the field generated in the plasma at the frequencies $2n\omega$ we have the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial \vec{r}^{\,2}} + \omega_L^2\right) \vec{E}_{2n}(\vec{r}, t) = - 4\pi \frac{\partial}{\partial t} \delta \vec{j}_{2n} \cos\left[2n(\omega t - \vec{k} \cdot \vec{r})\right], \quad (17)$$

where in the r.h.s. appears the current source, described by equation (11). As $\omega t \gg 1$, in solving equation (17) we can neglect the slow time variation of the current source amplitude. As a result, for the even harmonics electric field we find

$$\vec{E}_{2n}(\vec{r}, t) = -\vec{E}_{2n} \cos\left[2n(\omega t - \vec{k} \cdot \vec{r})\right]. \quad (18)$$

Using the dispersion relation (1) and the relations (11), (12), (18) from equation (17) we obtain the field strength $\vec{E}_{2n}$. This field defines the energy flux density at frequency $2n\omega$, $I_{2n} = cE_{2n}^2/8\pi$, and, at the same time, the even harmonics generation efficiency

$$\eta_{2n} = \frac{I_{2n}}{I} = \left(\frac{\nu}{\omega}\right)^2 \left(\frac{eE_0 t}{m v_T}\right)^2 H\left(2n, \gamma\right), \quad (19)$$

$$H\left(2n, \gamma\right) = \frac{2}{\pi} \frac{1}{\left(4n^2 - 1\right)^2} \gamma^{-2} \left[\alpha_l^2 G_l^2(n, \gamma) + \alpha_t^2 G_t^2(n, \gamma)\right], \quad (20)$$

where use is made of the notations $\alpha_l = \vec{\varepsilon} \cdot \vec{E}_0/E_0$, $\alpha_t = \vec{\kappa} \cdot \vec{E}_0/E_0$, while the functions G_l and G_t have the form

$$G_l(n, \gamma) = 2 \int_0^1 dx x^2 F\left[n, \frac{1}{4} x^2 \gamma^2\right], \quad (21)$$

$$G_t(n, \gamma) = \int_0^1 dx (1 - x^2) F\left[n, \frac{1}{4} x^2 \gamma^2\right]. \quad (22)$$

According to (19) even harmonics generation efficiency grows proportionally to t^2. It is traced back to the circumstance that, for a time interval $t < \nu^{-1}$, the velocity of electron directed motion, which determines the EDF degree of anisotropy, under the $\vec{E}_0$ action grows proportionally to t. Thanks to the inequality $\omega t \gg 1 > \nu^{-1}$, the harmonics electric field adiabatically follows the changes of the EDF degree of anisotropy. According to equation (20), the even harmonics generation efficiency depends on the direction of the constant electric field $\vec{E}_0$ with respect to the $\vec{E}$ and $\vec{k} \times \vec{E}$ vectors. It is due to the fact that the plasma conductivity has different values along the $\vec{E}$ and $\vec{k} \times \vec{E}$ vectors. As a consequence, in the plane orthogonal to the $\vec{k}$ vector the field $\vec{E}_{2n}$ undergoes rotation with respect to the field $\vec{E}$. The value of the rotation angle depends on the ratio v_E/v_T, the orientation of the vector $\vec{E}_0$ with respect to $\vec{E}$ and $\vec{k} \times \vec{E}$, the harmonic number n, and is given by the relation

$$\Psi(2n, \gamma) = \operatorname{arctg}\left[\frac{\alpha_t G_t(n, \gamma)}{\alpha_l G_l(n, \gamma)}\right]. \quad (23)$$

We note that no polarization rotation takes place in the case of odd harmonics. The functions η equations (15), (19) and Ψ (23) define the generation characteristics of high-frequency field harmonics in a *given plasma point* vs. the plasma and fields parameters. The radiation at harmonic frequencies going out from the plasma depends on the given specific experiment geometry. Discussion of the harmonics generation aspects due to the experimental specificity is out of the scope of the present paper.

4 Weak and strong field harmonics generation asymptotics

In the limits of weak and strong high-frequency fields the laws governing the high-order harmonics generation have a relatively simple form. We report below on the asymptotic formulae for even harmonics. In the weak field case, when $\gamma \ll 2\sqrt{n}$, from equations (12), (20–23) we find

$$H\left(2n, \gamma\right) \simeq \frac{2}{\pi} \gamma^{4n-2} \frac{\left[\left(2n+1\right)^2 \alpha_l^2 + \alpha_t^2\right]}{\left(2n+3\right)^2 (n!)^2 \left(4n^2-1\right)^2 4^{3n-1}}, \quad (24)$$

$$\Psi(2n, \gamma) \simeq \operatorname{arctg}\left[\frac{\alpha_t}{\alpha_l\left(2n+1\right)}\right], \quad \gamma \ll 2\sqrt{n}. \quad (25)$$

According to equation (24), even harmonics generation efficiency rapidly decreases with the increase of harmonic number. Besides, as it is seen from equation (25), diminishes also the departure of the harmonic field direction from that of the fundamental field. The most high values of generation efficiency correspond to the case when the constant electric field is along the fundamental high-frequency field. If this two fields are aligned $\alpha_l = 1$, $\alpha_t = 0$ and all the even harmonics have the same polarization as the fundamental field $\Psi(2n, \gamma) = 0$.

In the strong field case, when $\gamma \gg 2\sqrt{n}$, the relations (12), (20–23) yield the following asymptotic expressions

$$H\left(2n, \gamma\right) \simeq \frac{8}{\pi} \gamma^{-4} \frac{1}{\left(4n^2-1\right)^2} \left\{\alpha_t^2 \left[C_t - \gamma^{-2} L\left(n, \gamma\right)\right]^2 + 4\alpha_l^2 \gamma^{-4} \left[L\left(n, \gamma\right)\right]^2\right\}, \quad (26)$$

$$\Psi(2n, \gamma) \simeq \operatorname{arctg}\left\{\frac{\alpha_t}{2\alpha_l}\left[C_t \gamma^2 L^{-1}\left(n, \gamma\right) - 1\right]\right\}, \quad \gamma \gg 2\sqrt{n}, \quad (27)$$

where are used the notations $C_t \simeq 0.4$,

$$L\left(n, \gamma\right) = \frac{\left(4n^2-1\right)}{\sqrt{8\pi}} \left[\ln\left(\frac{\gamma^2}{4n}\right) + C(n)\right], \quad (28)$$

$C(1) \simeq 1.4$, $C(2) \simeq -0.33$, $C(3) \simeq -0.88$, $C(4) \simeq -1.2$, $C(5) \simeq -1.4, ...$

In a strong field, even harmonics generation efficiency decreases vs. the harmonic number n more slowly as compared with the weak field case. Efficiency generation decreases also with the increase of the ratio $\gamma = v_E/v_T$. The physical reason of such a behavior is the decreasing of the effective electron-ion collision frequency in a strong field. The rapidity of the function $H(2n, \gamma)$ decreasing vs. the increase of $\gamma = v_E/v_T$ depends on the $\overrightarrow{E}_0$ field direction with respect to the $\overrightarrow{E}$ and $\overrightarrow{k} \times \overrightarrow{E}$ vectors. The highest values of even harmonics generation efficiency result when $\overrightarrow{E}_0 || \overrightarrow{k} \times \overrightarrow{E}$. If instead the $\overrightarrow{E}_0$ field is directed along the strong high-frequency field, then the generation efficiency values are smaller than in the previous geometry by the small factor $\gamma^{-4}(\ln \gamma^2)^2 \ll 1$. The reason of the strong anisotropy in even harmonics generation efficiency is again traced back to the strong anisotropy of the effective electron-ion collision frequency in the presence of a strong high-frequency field. It is in fact known [9–11] that in the direction orthogonal to the $\overrightarrow{E}$ field the effective collision frequency becomes larger than in the parallel direction by the factor $\gamma^2/\ln\gamma^2 \gg 1$. The effect of the collision frequency strong anisotropy manifests itself also in the behavior of the function $\Psi(2n, \gamma)$, equation (27). In fact, when $\overrightarrow{E}_0 || \overrightarrow{k} \times \overrightarrow{E}$ the rotation angle of the even harmonics field with respect to the fundamental field has its largest value, being $\pi/2$. Instead, if $\overrightarrow{E}_0 || \overrightarrow{E}$, there is no rotation effect and $\Psi(2n, \gamma) = 0$.

We conclude this section reporting asymptotic expressions for two limiting cases for the function $H(2n+1, \gamma)$ equation (16) as well. For $\gamma \ll 2\sqrt{n}$, from equation (16) we have

$$H(2n+1, \gamma) \simeq \frac{8}{\pi} \left[2^n (n+1)!\, n\, (2n+3)\right]^{-2} \left(\frac{\gamma}{2}\right)^{4n}, \quad \gamma \ll 2\sqrt{n}, \quad (29)$$

while for $\gamma \gg 2\sqrt{n}$, equation (16) gives

$$H(2n+1, \gamma) \simeq \gamma^{-6} \left[\frac{(2n+1)}{\pi n(n+1)} \left(\ln \frac{\gamma}{2\sqrt{n}} - C_n\right)\right]^2, \quad \gamma \gg 2\sqrt{n}, \quad (30)$$

where $C_1 = 0.31$, $C_2 = 0.50$, $C_3 = 0.64$, $C_4 = 0.75$, $C_5 = 0.84$.

The asymptotic expressions reported in this section are meant to help to visualize the specific roles of the basic radiation and plasma parameters in the two opposite limits of weak and strong fields. They are also useful and complementary when dealing with the numerical evaluation of $H(2n, \gamma)$ and $\Psi(2n, \gamma)$.

5 Calculations of the harmonics basic characteristics

The analytical relations derived above are illustrated by numerical calculations reported in Figures 1–5.

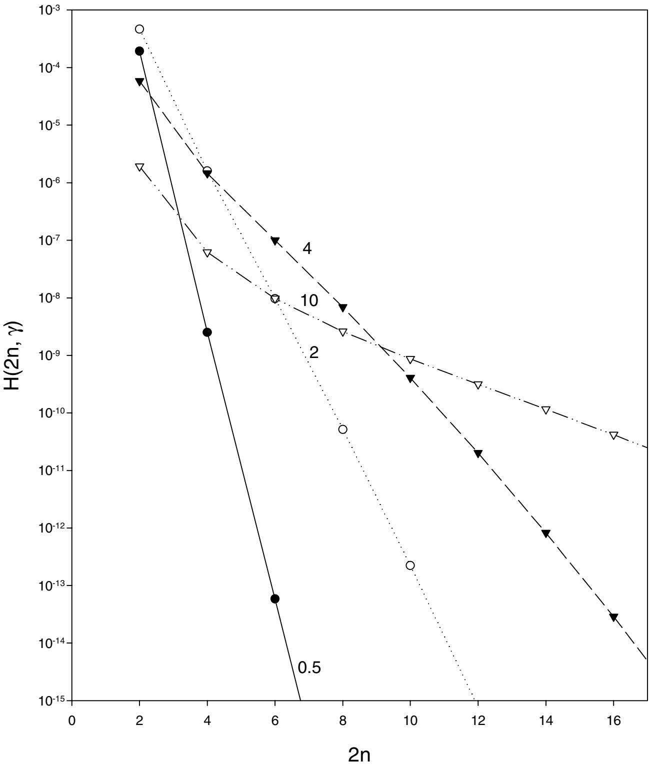

Fig. 1. Even harmonics generation efficiency vs. the harmonic number for $\alpha_t = \alpha_l = 1/\sqrt{2}$, with α_l and α_t being, respectively, cosines of the angles between $\overrightarrow{E_0}$ and $\overrightarrow{E}$ or $\overrightarrow{k} \times \overrightarrow{E}$. The different sets of discrete points are connected by lines to help visualization. The numbers on the curves correspond to four values of the ratio of the electron quiver velocity to the thermal one $\gamma = v_E/v_T = 0.5$, 2, 4, 10.

In Figure 1 is shown the $H(2n, \gamma)$ function (20) dependence on the harmonic number for different values of the parameter γ. For a given γ the points corresponding to different n are connected by a line to help visualization. Calculations are performed for $\alpha_l = \alpha_t = 1/\sqrt{2}$, when the field $\overrightarrow{E}_0$ forms the angle $\pi/4$ with either $\overrightarrow{E}$ and $\overrightarrow{k} \times \overrightarrow{E}$. According to Figure 1, for a given $\gamma = v_E/v_T$ the generation efficiency decreases with the increase of the harmonic number. This behavior is also seen in formulae (24), (26).

Fixing the harmonic number and increasing γ the function $H(2n, \gamma)$ initially grows, around $\gamma \approx 2\sqrt{n}$ reaches its maximum value and then monotonically decreases. It is easily seen by comparing points from different lines corresponding to a given n. The non-monotonical behavior of $H(2n, \gamma)$ vs. γ is shown in Figure 2, where $H(2n, \gamma)$ is plotted for the second, forth and sixth harmonics ($n = 1$, 2 and 3). For these harmonics Figure 2 shows that the generation efficiency largest values are reached respectively at $\gamma(2\omega) \approx 1.8$, $\gamma(4\omega) \approx 2.8$ and $\gamma(6\omega) \approx 4.0$. All the results plotted in Figure 2 are obtained for $\alpha_l = \alpha_t = 1/\sqrt{2}$. For others α_l and α_t values, the dependencies are similar, with relatively small numerical departures. We remark that the presence of a maximum of the function $H(2n, \gamma)$ around

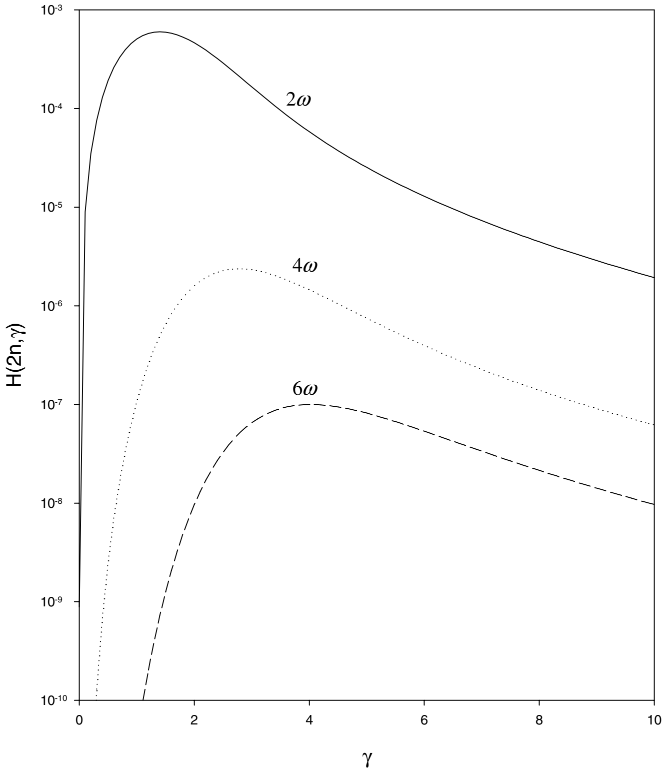

Fig. 2. The same as Figure 1 vs. $\gamma = v_E/v_T$ for $\alpha_t = \alpha_l = 1/\sqrt{2}$. The numbers on the curves are, respectively for the second, forth and sixth harmonics. For the meaning of symbols see caption to Figure 1.

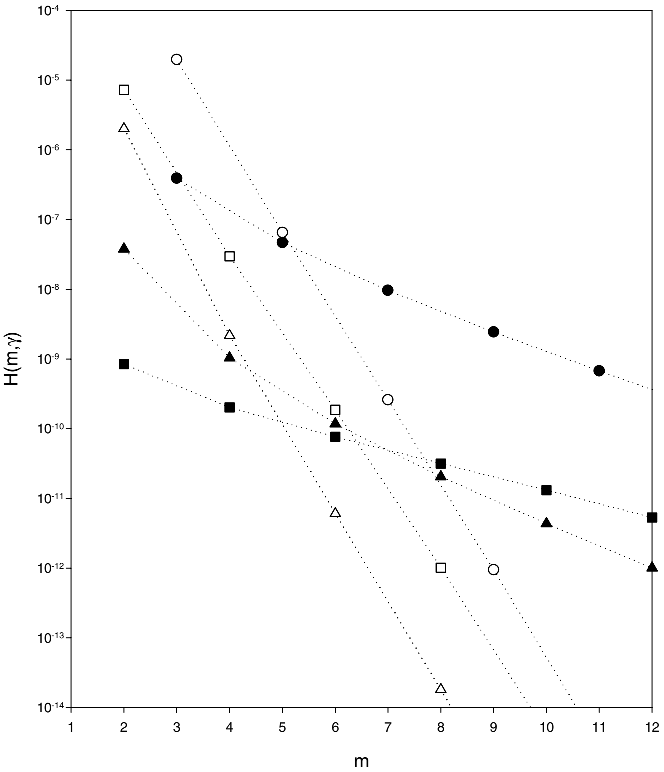

Fig. 3. Harmonics generation efficiency $H(m, \gamma)$ vs. the harmonic number m for $\gamma = 2$ (white symbols) and 10 (black symbols). To each γ value correspond two sets of points with even m numbers, obtained for $\alpha_l = 0$, $\alpha_t = 1$ (triangles) and $\alpha_l = 1$, $\alpha_t = 0$ (squares). Odd harmonics are marked by circles. For the meaning of symbols see caption to Figure 1.

$\gamma \approx 2\sqrt{n}$ may be established also from the asymptotic relations (24), (26).

Figure 3 reports the dependence of $H(m, \gamma)$ vs. m for $\gamma = 2$ and 10. m is either $2n$ on $2n + 1$. For each γ value are reported two sets of points corresponding to $\alpha_l = 0$, $\alpha_t = 1$ and to $\alpha_l = 1$, $\alpha_t = 0$. From Figure 3 and formulae (24), (26), (29) and (30) the following characteristic features may be singled out. First, in the strong field case, when $\gamma \gg 2\sqrt{n}$, and at fixed γ value, the even harmonics generation efficiency has its largest value for $\alpha_l = 0$, $\alpha_t = 1$, i.e. when $\overrightarrow{E}_0 || \overrightarrow{k} \times \overrightarrow{E}$. At the contrary, in the weak field case, when $\gamma \ll 2\sqrt{n}$, even harmonics are generated better if $\alpha_l = 1$, $\alpha_t = 0$, i.e. when $\overrightarrow{E}_0 || \overrightarrow{E}$. These properties may be deduced from formulae (24), (26) as well, and are explained by the anisotropy of the electron effective collision frequency in a strong high-frequency field. Second, even harmonics generation efficiency may be as high as that of the odd ones, and even higher, in spite of the presence, in its defining expression (19), of the small factor $(eE_0t/mv_T)^2 \ll 1$, (see Fig. 3). Figure 4 shows the function $\Psi(2n, \gamma)$ dependence on n, for the same γ values as in Figure 1. According to Figure 4, than larger γ, than greater the rotation angle the even harmonics polarization plane forms with respect to that of the fundamental field. For a given γ value the function $\Psi(2n, \gamma)$ monotonically decreases with the increase of the harmonic number. Figure 5 shows instead the $\Psi(2n, \gamma)$ monotonical growth with the γ values increase. Here are plotted the $\Psi(2n, \gamma)$ curves for the second, forth and sixth harmonics. From Figure 5 the $\Psi(2n, \gamma)$ decrease vs. n may be seen as well.

Let us give an estimate of the even harmonics generation efficiency. Let the frequency of the fundamental radiation be $\omega = 2 \times 10^{14}$ s^{-1}, and the energy flux density $I = 6 \times 10^{12}$ W/cm^2, and let the plasma parameters be $Z = 2$, $T = 50$ eV and $N = 10^{18}$ cm^{-3}. With these radiation and plasma parameters the effective e-i collision frequency is $\nu = 4 \times 10^{11}$ s^{-1}, the electron plasma frequency $\omega_L = 6 \times 10^{13}$ s^{-1} and the ratio $\gamma = v_E/v_T \simeq 2$. Let us assume that the constant field $\overrightarrow{E}_0$ direction form the angle $\pi/4$ with both the vectors $\overrightarrow{E}$ and $\overrightarrow{k} \times \overrightarrow{E}$, which gives $\alpha_t = \alpha_l = 1/\sqrt{2}$. Taking the strength of the constant electric field $E_0 = 10$ kV/cm, in the time interval $t < \nu^{-1} \simeq 2$ ps the electron drift velocity remains small as compared to v_T, and at the instant time $t \simeq 2$ ps reaches the value $|eE_0t/m| \simeq 0.1v_T$. Within the chosen conditions, in accordance with formulae (19), (20) and Figure 2, the second harmonic generation efficiency is $\eta(2\omega) \simeq 2 \times 10^{-11}$, corresponding to the energy flux density $I(2\omega) \simeq 100$ W/cm^2. The forth harmonic generation efficiency is smaller than $\eta(2\omega)$ by two orders of magnitude.

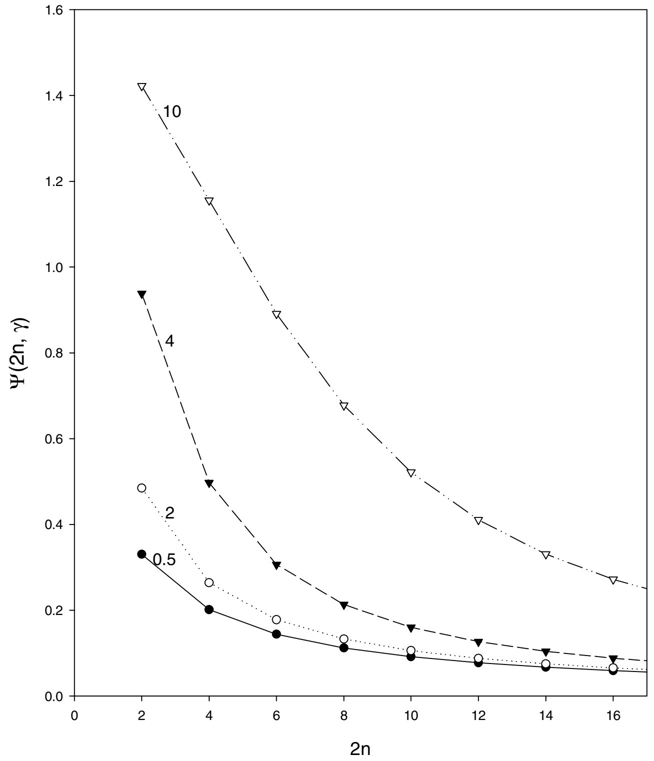

Fig. 4. The rotation angle $\Psi(2n, \gamma)$ of the even harmonics polarization plane with respect to that of the fundamental wave vs. the harmonic number. The numbers on the curves correspond to four values of the ratio of the electron quiver velocity to the thermal one $\gamma = v_E/v_T = 0.5$, 2, 4, 10. For the meaning of symbols see caption to Figure 1.

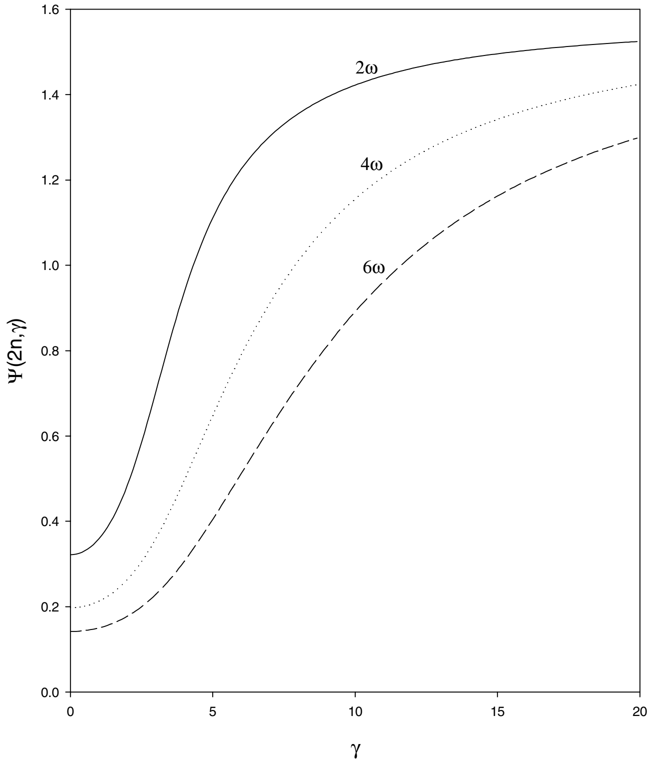

Fig. 5. The same function as in Figure 4 vs. $\gamma = v_E/v_T$ for the even harmonics 2ω, 4ω and 6ω. For the meaning of symbols see caption to Figure 1.

6 Conclusions

We have shown that the electron directed motion due to the presence of a constant electric field yields an enrichment of plasma emission spectrum if the plasma interacts with a high-frequency external radiation field. Specifically, in the plasma spectrum appear even harmonics of the external high-frequency field. This property may be exploited to diagnose electron drift motions in plasmas. In fact, the established properties of the generated even harmonics allow to diagnose in short time intervals how a constant homogeneous electric field acts on a plasma. At the same time, it is clear that a similar possibility occur also when the time of the constant field action is longer than the inverse of the e-i effective collision frequency. For times greater than ν^{-1} in a static field $\overrightarrow{E}_0$ a quasistationary electron drift is established, if E_0 is smaller than the Dreicer critical field. In such a case, the EDF anisotropic part is inversely proportional to the e-i collision frequency and exhibits a dependence on velocity, different as compared to that occurring at small times, when $\nu t < 1$. As a consequence, the even harmonic properties for $\nu t > 1$ are different as compared to those resulting when $\nu t < 1$. The investigation of the harmonic properties in the case $\nu t > 1$ requires a somewhat different treatment, which is dealt with in a separate paper [12].

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